

Localization of spin-waves in disordered quantum rotors

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We study the dynamics of excitations in a system of $O(N)$ quantum rotors in the presence of random fields and random anisotropies. Below the lower critical dimension $d_{lc} = 4$ the system exhibits a quasi-long-range order with a power-law decay of correlations. At zero temperature the spin-waves are localized at the length scale L_{loc} beyond which the quantum tunneling is exponentially suppressed $c \sim e^{-(L/L_{loc})^{2(\theta+1)}}$. At finite temperature T the spin-waves propagate by thermal activation over energy barriers that scales as L^θ . Above d_{lc} the system undergoes an order-disorder phase transition with activated dynamics such that the relaxation time grows with the correlation length ξ as $\tau \sim e^{C\xi^\theta/T}$ at finite temperature and as $\tau \sim e^{C'\xi^{2(\theta+1)}/\hbar^2}$ in the vicinity of the quantum critical point.

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Introduction. – Localization of excitations in disordered quantum systems has been attracting considerable interest during the last several decades. While the single particle localization is rather well understood within the standard theory of Anderson localization [1], the localization of interacting particles is a much more complicated problem where many questions remain open [2–4]. Excited many-body localized fermionic eigenstates were studied in the random field Heisenberg spin- $\frac{1}{2}$ chain using exact diagonalization [5] and in the random anisotropy XXZ spin- $\frac{1}{2}$ chain applying a dynamical real space renormalization group [6]. It was found that the many-body localized states in closed quantum systems with quenched randomness share many properties of quantum glasses, *e.g.* they fail to thermally equilibrate and breaks ergodicity. It was argued that they can be described by an infinite-randomness fixed point (FP) with an infinite dynamic critical exponent [4–6]. Unlike fermions, bosons can condense into a superfluid state with long-range order so that interactions are intrinsically unavoidable. The presence of disorder can suppress the phase coherence of the bosons and localize them collectively in the so-called Bose glass with a gapless energy spectrum and a finite compressibility [7] or into the incompressible Mott glass [8]. The zero temperature superfluid-insulator transition in two-dimensional disordered hard-core bosons has been recently studied using a spin-wave approach [9]. A mobility edge in the spin-wave excitation spectrum has been found at a finite frequency which vanishes in the Bose glass phase. The connection between the Bose (Mott) glass and disordered elastic systems has been known for long time [10–14]. Recently, a mapping of the leading order perturbation theory for boson Green's functions to a directed polymer in random media has been proposed for studying the insulating phase of charged hard-core bosons [15].

In this paper we investigate the dynamics of a d -dimensional system of $O(N)$ quantum rotors in the presence of random fields and random anisotropies. This model shares many properties with the aforementioned systems but allows

for an analytical study using functional renormalization group (FRG) originally developed for disordered elastic systems such as the directed polymer in random media [16, 17]. The FRG reveals that the behavior of the disordered quantum rotors is controlled by a quasi-classical zero temperature FP. In the real space renormalization group treatment of spin chains one fixes the temperature and the Planck constant so that the renormalized disorder strength grows approaching an infinite-randomness FP. In our FRG scheme we fix the disorder strength near the FP but allow the temperature and the effective Planck constant to flow to zero. The both parameters turn out to be dangerously irrelevant like the temperature in the random field Ising model [18]. This drastically changes the dynamic scaling picture which one could expect from a naive RG treatment [19, 20]. Appearance of non-analyticity in the FRG flow prevents the system from equilibration by inducing the activated dynamics with diverging barriers at finite temperature and localization at zero temperature. This mechanism is to some extent similar to the one behind the classical and quantum creep of disordered elastic systems at small driving forces [21, 22].

Model. – The Hamiltonian of interacting quantum rotors on a d -dimensional hyper-cubic lattice with lattice constant b can be written as

$$\mathcal{H}_0 = \frac{1}{2I} \sum_i \hat{\mathbf{L}}_i^2 - \sum_{\langle i,j \rangle} J_{ij} \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j, \quad \hat{\mathbf{n}}_i^2 = 1, \quad (1)$$

where the operator $\hat{\mathbf{n}}_i$ is a N -dimensional unit-length vector representing the orientation of the rotor sitting on site i . $\hat{\mathbf{L}}_i$ is the angular momentum operator whose $N(N-1)/2$ components are defined as $\hat{L}_{i\mu\nu} = \hat{n}_{i\mu}\hat{p}_{i\nu} - \hat{n}_{i\nu}\hat{p}_{i\mu}$ and the momentum operator of each rotor satisfies the commutation relations $[\hat{n}_{i\mu}, \hat{p}_{j\nu}] = i\hbar\delta_{ij}\delta_{\mu\nu}$. The first term in (1) is the kinetic energy of the rotor with the moment of inertia I . In the case of randomly distributed exchange interactions J_{ij} the system forms a strong quantum glass which has been studied mainly in the limit of infinite range interactions using $1/N$ -

expansion [23]. The limit of $N = 1$ is expected to be in the same universality class as the Ising model in a transverse field whose glass phase is critical everywhere and exhibits gapless collective excitations in the long-range interaction limit [24]. Here we assume that all $J_{ij} = J$ and restrict the sum $\langle i, j \rangle$ to nearest neighbors. Instead of the random exchange interactions we introduce random fields and random anisotropies: $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{RF}} + \mathcal{H}_{\text{RA}}$, where $\mathcal{H}_{\text{RF}} = -\sum_i \mathbf{h}_i \cdot \hat{\mathbf{n}}_i$ and $\mathcal{H}_{\text{RA}} = -\sum_i (\mathbf{d}_i \cdot \hat{\mathbf{n}}_i)^2$ with randomly oriented vectors \mathbf{h}_i and \mathbf{d}_i .

In the continuum limit the model (1) can be rewritten as an $O(N)$ quantum-mechanical nonlinear σ -model (QNL σ M) with the partition function $\mathcal{Z} = \int \mathcal{D}\mathbf{n} \delta(|\mathbf{n}| - 1) e^{-S[\mathbf{n}]/\hbar}$ and the imaginary time action

$$S[\mathbf{n}] = \frac{\rho_0}{2} \int_{\tau, x} \left[\frac{1}{c_0^2} (\partial_\tau \mathbf{n}(\tau, x))^2 + (\nabla \mathbf{n}(\tau, x))^2 \right] - \int_{\tau, x} \sum_{\mu=1}^{\infty} \sum_{i_1 \dots i_\mu} h_{i_1 \dots i_\mu}^{(\mu)}(x) n_{i_1}(\tau, x) \dots n_{i_\mu}(\tau, x), \quad (2)$$

where we have introduced the shortcut notations $\int_\tau := \int_0^{\hbar/T} d\tau$ and $\int_x := \int d^d x$. Here $\rho_0 = b^{2-d} J$ is the bare stiffness constant, $c_0 = b\sqrt{J/T}$ - the bare spin-wave velocity and T - the temperature. The UV cutoff $\Lambda_0 = 2\pi/b$ is imposed in (2). The $O(N)$ QNL σ M arises as an effective theory for the low energy degrees of freedom in several correlated quantum systems. For instance, the $O(2)$ model could describe Cooper pairs of electrons in a superconducting Josephson junctions array and ultra-cold atoms in an optical lattice [25]. The $O(3)$ model describes a quantum spin- S antiferromagnet in the large- S limit [26]. The $O(5)$ QNL σ M was suggested for the unified low-energy theory of the antiferromagnetic and superconducting phases in the high- T_c superconductors [27]. The renormalization of the original model (1) generates the higher rank anisotropies, which we incorporated in the second line of (2) from the beginning: $h_i^{(1)}$ is the random field, $h_{ij}^{(2)}$ - the random second-rank anisotropy, and $h^{(\mu)}$ - random μ th rank anisotropy [28, 29]. The RG flow preserves the symmetry with respect to inversion $\mathbf{n} \rightarrow -\mathbf{n}$, so we will use the notation of random anisotropy (RA) for the systems respecting this symmetry and random field (RF) for the rest. The bare μ th rank anisotropies can be taken Gaussian distributed with zero mean and cumulants

$$\overline{h_{i_1 \dots i_\mu}^{(\mu)}(x) h_{j_1 \dots j_\nu}^{(\nu)}(x')} = \delta^{\mu\nu} \delta_{i_1 j_1} \dots \delta_{i_\mu j_\nu} r^{(\mu)} \delta(x - x'). \quad (3)$$

We use replica trick to average over disorder. Introducing n replicas of the original system we obtain the replicated action

$$\mathcal{S}_n[\{\mathbf{n}\}] = \frac{\rho_0}{2} \sum_{a=1}^n \int_{\tau, x} \left[\frac{1}{c_0^2} (\partial_\tau \mathbf{n}_a(\tau, x))^2 + (\nabla \mathbf{n}_a(\tau, x))^2 \right] - \frac{1}{2\hbar} \sum_{a,b=1}^n \int_{\tau, \tau'} R(\mathbf{n}_a(\tau, x) \cdot \mathbf{n}_b(\tau', x)), \quad (4)$$

where we have introduced $R(z) = \sum_\mu r^{(\mu)} z^\mu$, which is defined for $-1 \leq z \leq 1$. This function is even for the RA model and has no symmetry for the RF model. The properties of the original disordered system (2) can be extracted in the limit $n \rightarrow 0$. The Imry-Ma arguments suggest that the true long-range order is absent in our model for $d < d_{lc} = 4$. However, a quantum QLRO can survive at low enough temperature similarly to the QLRO in the classical Heisenberg model [30]. In the QLRO phase the local order slowly changes in space leading to a power-law decay of correlations that justifies the description of the dynamics in terms of spin-wave excitations.

FRG. - To get access to the quantum glassy phase we renormalize the action (4) using a momentum-shell method in which iterative integrations over fast modes with wavevectors between the bare cutoff Λ_0 and the running cutoff $\Lambda_\ell = \Lambda_0 e^{-\ell}$ generate the RG flow equations. It is convenient to express them in terms of the reduced running quantities: $\tilde{R}_\ell(\phi) = K_d R_\ell(z) \rho_\ell^{-2} \Lambda_\ell^{d-4}$ where $z = \cos \phi$, $\tilde{h}_\ell = K_d \hbar \rho_\ell^{-1} \Lambda_\ell^{d-1}$, and $\tilde{T}_\ell = K_d T \rho_\ell^{-1} \Lambda_\ell^{d-2}$, where K_d is the surface of the unit sphere in d - dimensions divided by $(2\pi)^d$. The function $\tilde{R}(\phi)$ is π -periodic for the RA and 2π -periodic the RF models. We expand around a locally ordered state and neglect the possible presence of topological defects which can modify the behavior of the system [31]. We split the local order parameter $\mathbf{n}_a = (\sigma_a, \boldsymbol{\pi}_a)$ into component $\sigma_a = \sqrt{1 - \boldsymbol{\pi}_a^2}$ aligned along the locally preferred direction and the $(N-1)$ -component vector $\boldsymbol{\pi}_a$ perpendicular to it. We decompose the latter in slowly and rapidly varying parts $\boldsymbol{\pi}_a^<$ and $\boldsymbol{\pi}_a^>$ with the momentum modes $0 < q < \Lambda_\ell$ and $\Lambda_\ell < q < \Lambda_0$, respectively. Integrating out the fast fields $\boldsymbol{\pi}_a^>$ and allowing for the rescaling of the slow fields $\boldsymbol{\pi}_{aR}(x) = \zeta \boldsymbol{\pi}_a^<(x)$ with $\zeta = 1 + \frac{1}{2}(N-1)\tilde{R}_\ell''(0)\ell + O(\tilde{R}_\ell^2)$, we obtain the one-loop flow equations for the effective temperature and the Planck constant

$$\partial_\ell \ln \tilde{T}_\ell = 1 + \partial_\ell \ln \tilde{h}_\ell = 2 - d - (N-2)\tilde{R}_\ell''(0), \quad (5)$$

and for the disorder correlator

$$\partial_\ell \tilde{R}_\ell(\phi) = \varepsilon \tilde{R}_\ell(\phi) + \tilde{R}_\ell'(\phi) [\Gamma_\ell - \tilde{R}_\ell''(0)] + \frac{1}{2} [\tilde{R}_\ell''(\phi)]^2 + (N-2) \left(\frac{\tilde{R}_\ell'(\phi)^2}{2 \sin^2 \phi} + \left[\frac{\tilde{R}_\ell'(\phi)}{\tan \phi} + 2\tilde{R}_\ell(\phi) \right] [\Gamma_\ell - \tilde{R}_\ell''(0)] \right). \quad (6)$$

Here we defined $\varepsilon = 4 - d$ and the boundary layer width

$$\Gamma_\ell = \frac{1}{2} c_\ell \tilde{h}_\ell \coth \left[\frac{c_\ell \tilde{h}_\ell}{2\tilde{T}_\ell} \right] = \begin{cases} \tilde{T}_\ell & \text{if } \tilde{h}_\ell \rightarrow 0, \\ \frac{1}{2} c_\ell \tilde{h}_\ell & \text{if } \tilde{T}_\ell \rightarrow 0, \end{cases} \quad (7)$$

which describes the joint effect of thermal and quantum fluctuations on the disorder correlator flow. Disorder breaks the Lorentz invariance and renormalizes the spin-wave velocity c_0 like the stiffness constant in disordered elastic systems with broken statistical tilt symmetry [32]. Its flow equation reads

$$\partial_\ell \ln c_\ell = -\frac{1}{6} \left[(N+1)\tilde{R}_\ell^{(4)}(0) + (N-2)\tilde{R}_\ell''(0) \right]. \quad (8)$$

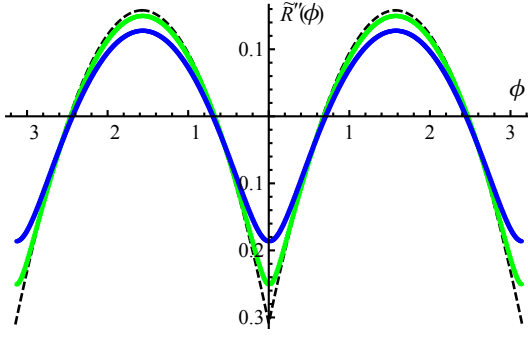


FIG. 1. (Color online) The π -periodic FP solution $\tilde{R}''(\phi)$ describing the QLRO phase in the 3D $O(3)$ RA model. The dashed line is the zero- Γ FP, the solid green and blue lines with a rounded cusp are the finite Γ FPs for $\Gamma = 0.05$ and $\Gamma = 0.1$.

Assuming that the running disorder correlator reaches an attractive FP of the flow equation (6) one might naively conclude from (5) and (8) that the system exhibits a usual critical scaling behavior. However, the more accurate analysis shows that this is not the case.

Localization and activated dynamics. – We start the analysis of the equations (5)-(7) in $d < 4$ by studying the flow of the disorder correlator for $\Gamma_\ell = 0$, *i.e.* neglecting thermal and quantum fluctuations. For concreteness we take a smooth π -periodic correlator $\tilde{R}_0(\phi) = \gamma \cos^2 \phi$ (the RA universality class). The flow equations for the first derivatives of $\tilde{R}_\ell(\phi)$ at $\phi = 0$ that follow from (6) imply that the renormalized $\tilde{R}_\ell^{(4)}(0)$ diverges at the finite scale $\ell_c \approx \frac{1}{\varepsilon} \ln(1 + 3\varepsilon/[8\gamma(N+7)])$. Beyond this scale the running disorder correlator becomes non-analytic at $\phi = 0$: the second derivative develops a cusp, $\tilde{R}_\ell'''(0^+) \neq 0$ for $\ell > \ell_c$. Then the renormalized disorder correlator $\tilde{R}_\ell(\phi)$ rapidly approaches a non analytic FP solution $\tilde{R}^*(\phi)$ with $\tilde{R}'''(0^+) \neq 0$ and finite $\tilde{R}^{(4)}(0^+)$. The stable non-analytic FP solution exists for $2 \leq N \leq N_c$. To one-loop order $N_c = 2.835$ for RF and $N_c = 9.441$ for RA are close to their classical limits [33–35]. For instance, the $O(2)$ models have the FPs with $\tilde{R}'''(0) = -\phi_0^2\varepsilon/36$, where $\phi_0 = \pi$ and $\phi_0 = 2\pi$ for the RA and RF, respectively. The $O(3)$ and $O(4)$ RA models have the FPs with $\tilde{R}'''(0) \approx -0.309\varepsilon$ and $\tilde{R}'''(0) \approx -0.358\varepsilon$. The numerical zero- Γ RA FP solution for $N = 3$ is shown in Fig. 1.

The numerical solution of the full FRG flow shows that the running disorder correlator can be approximated for $\ell > \ell_c$ by the FP point solution of the flow equation (6) at fixed Γ_ℓ . At finite but small Γ_ℓ this FP solution uniformly approaches the zero- Γ FP solution everywhere except for the extreme points (see Fig. 1). The physically most relevant region is the boundary layer around $\phi = 0$ which has the width of order Γ_ℓ . Within the boundary layer the cusp of the zero- Γ FP solution $\tilde{R}'''(\phi)$ is rounded by thermal and quantum fluctuations. Indeed, since Γ_ℓ flows towards zero the second derivative $\tilde{R}_\ell''(0)$ approaches $\tilde{R}''(0) \neq 0$ while $\tilde{R}_\ell^{(4)}(0)$ diverges, and thus, remains different from $\tilde{R}^{(4)}(0^+)$ for arbitrary small but fi-

nite Γ_ℓ . This results in activated dynamic scaling similar to that found in the random transverse field Ising model [36, 37] and may lead to different behavior of averaged and typical correlations and multifractality [38]. In particular the averaged connected and disconnected correlations scale differently: $G_{\text{con}}(x) \sim 1/x^{d-2+\eta}$ and $G_{\text{dis}}(x) \sim 1/x^{d-4+\bar{\eta}}$, with the exponents $\eta = -\tilde{R}''(0)$ and $\bar{\eta} = \varepsilon - (N-1)\tilde{R}''(0)$ that can be extracted from the rescaling factor ζ at the FP (see *e.g.* [35]). The algebraic decay of correlators implies that the spectrum of excitations remains gapless in the whole quantum QLRO phase. This is in contrast to the pure model in the disordered phase with a gap in the energy spectrum that vanishes only at the transition to the ordered state: the quantum transition occurs when the bare effective coupling constant $g_0 = c_0\tilde{h}_0$ crosses a nontrivial FP $g^* = 2(d-1)/(N-2)$ at zero temperature while the thermal transition takes place along the separatrix controlled by a thermal FP $g^* = 0$ and $\tilde{T}^* = (d-2)/(N-2)$ [26].

To find the flow of the disorder correlator in the boundary layer we expand the flow equation (6) in small ϕ for fixed Γ_ℓ . To lowest order in Γ_ℓ this gives $\tilde{R}_\ell''(0) \approx \tilde{R}''(0)$ and $\tilde{R}_\ell^{(4)}(0) \approx 6\Omega/[\Gamma_\ell(N+1)]$ with $\Omega = \frac{1}{2}\tilde{R}''(0)[\tilde{R}'''(0)(N-2) - \varepsilon]$. Neglecting the renormalization for $\ell < \ell_c$ we obtain from (5) that $\tilde{T}_\ell = \tilde{T}_0 e^{-\theta(\ell-\ell_c)}$ and $\tilde{h}_\ell = \tilde{h}_0 e^{-\theta_h(\ell-\ell_c)}$. The exponents θ and θ_h are given by $\theta = \theta_h - 1 = d-2 + (N-2)\tilde{R}''(0)$ to one loop order, but we conjecture that the relation $\theta_h = 1 + \theta$ holds to all orders. Substituting the disorder correlator derivatives in the boundary layer to the spin-wave velocity flow (8) and omitting the subdominant terms we find

$$\partial_\ell \ln c_\ell = -\frac{\Omega}{\Gamma_\ell}. \quad (9)$$

In the classical limit $\tilde{h} \rightarrow 0$, $\tilde{T} \rightarrow \infty$ the rounding of the cusp in the boundary layer is governed by thermal fluctuations, $\Gamma_\ell \approx \tilde{T}_\ell$. Neglecting renormalization of the spin-wave velocity c below the scale ℓ_c we arrive at

$$c_\ell = c_0 e^{-\frac{\Omega}{T_0\theta} [e^{\theta(\ell-\ell_c)} - 1]}. \quad (10)$$

Thus, in the classical regime the low frequency spin-waves propagate via thermal activation over energy barriers that grow with the length scale $L = \Lambda_0^{-1}e^\ell$ as L^θ . We believe that this result is also applicable to the classical $O(N)$ models with Langevin dynamics where c has to be replaced by the kinetic coefficient [39]. While early numerical works [40] confirmed a power-law decay of correlations in the classical $O(N)$ models, recent numerical simulations [31] suggested that the presence of topological defects can lead to an exponential decay of correlations on the scales larger than the average distance between the defects. Thus, there is a possibility for a scenario when the dynamics is described by (10) while the algebraic decay of correlations is screened by the topological defects whose relaxation time is very large. In the opposite limit of $T \rightarrow 0$, the spin-wave velocity flows as

$$c_\ell = c_0 + \frac{2\Omega}{\tilde{h}_0\theta_h}(1 - e^{\theta_h(\ell-\ell_c)}) \quad (11)$$

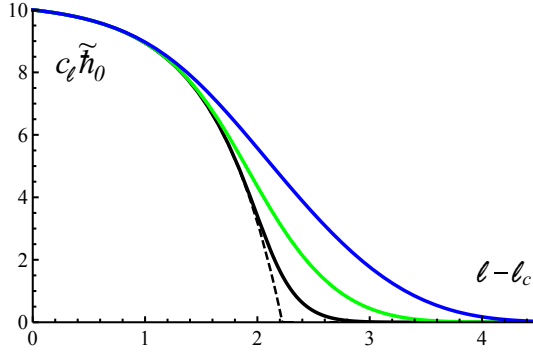


FIG. 2. (Color online) The renormalized spin-wave velocity in the 3D $O(3)$ RA model as a function of $\ell - \ell_c$ for different T and the initial condition for the bare coupling constant $g_0 = c_0 \tilde{\hbar}_0 = 10$. The dashed line corresponds to $\tilde{T}_0 = 0$; the solid black, green, and blue lines to $\tilde{T}_0 = 0.2; 0.5; 1$.

and vanishes at a finite length scale $L_{\text{loc}} = \Lambda_0^{-1} e^{\ell_{\text{loc}}}$ with

$$\ell_{\text{loc}} - \ell_c = \frac{1}{\theta_h} \ln \left[1 + \frac{c_0 \tilde{\hbar}_0 \theta_h}{2\Omega} \right]. \quad (12)$$

This scale can be interpreted as a $T = 0$ spin-wave localization length. The renormalized spin-wave velocity computed from numerical integration of the flow equation (9) for different temperatures is shown in Fig. 2. For finite but small temperature T one can define an effective localization length

$$L_T \approx L_{\text{loc}} \left[1 + \frac{\tilde{T}_0 \theta}{\Omega} \ln [c_0 \Lambda_0 \tau_{\text{exp}}] \right]^{1/\theta}, \quad (13)$$

beyond which the activated dynamics can be neglected on the time scale of experiment τ_{exp} .

The residual quantum tunneling. – We now show that taking into account the renormalization of the high frequency part of the excitation spectrum leads to an extremely small but finite spin-wave velocity in the low frequency limit even at zero temperature. To see that we generalize the bare part of the effective action (4) at $T = 0$ to

$$\mathcal{S}_n^{(0)} = \frac{\rho_0}{2} \sum_{a=1}^n \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^d q}{(2\pi)^d} [D(\omega) + q^2] |\mathbf{n}_a(\omega, q)|^2. \quad (14)$$

Such generalization does not modify the flow equations for the effective Planck constant (5) and disorder correlator (6). The zero temperature boundary layer width is, however, now given by

$$\Gamma_\ell = \frac{\tilde{\hbar}_\ell}{\Lambda_\ell} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{1 + \tilde{D}_\ell(\omega)}, \quad (15)$$

where we have defined $\tilde{D}_\ell(\omega) = \Lambda_\ell^{-2} D_\ell(\omega)$. The flow of the spectrum $\tilde{D}(\omega)$ starting from an arbitrary phonon-like spectrum $\tilde{D}_0(\omega)$ to one loop order reads

$$\partial_\ell \tilde{D}_\ell(\omega) = 2\tilde{D}_\ell(\omega) + \frac{2\Omega}{\Gamma_\ell} \frac{\tilde{D}_\ell(\omega)}{1 + \tilde{D}_\ell(\omega)}. \quad (16)$$

Here we have only retained the terms that are relevant in the limit $\Gamma_\ell \rightarrow 0$. The renormalized spectrum and the boundary layer width are solutions of the self-consistent equations (15) and (16). The previous derivation of the boundary layer width (7) assumed that it is determined exclusively by the low frequency part of the spectrum. This resulted in vanishing spin-wave velocity beyond the finite scale L_{loc} . However, renormalization of the high frequency part of the spectrum (16) gives a contribution to the boundary layer width (15) that leads to an exponentially small spin-wave velocity on scales $L > L_{\text{loc}}$. To see this we solve the spectrum flow equation (16) in the high frequency region $\tilde{D}(\omega) \gg 1$:

$$\tilde{D}_\ell(\omega) \approx e^{2\ell} \tilde{D}_0(\omega) + 2\Omega \int_{\ell_c}^{\ell} \frac{d\ell'}{\Gamma_{\ell'}} e^{2(\ell-\ell')}. \quad (17)$$

Plugging this in (15) and taking the bare spectrum as $\tilde{D}_0(\omega) = \omega^2/(c_0^2 \Lambda_0^2)$ we integrate out the high frequencies and obtain a Volterra-type integral equation for Γ_ℓ . For large ℓ it can be transformed into a differential equation:

$$\frac{d}{d\ell} \left[\frac{c_0 \tilde{\hbar}_\ell e^{-(\ell-\ell_c)}}{2\Gamma_\ell} \right]^2 = \frac{2\Omega}{\Gamma_\ell} e^{-2(\ell-\ell_c)}, \quad (18)$$

whose solution is $\Gamma_\ell = c_0^2 \tilde{\hbar}_\ell^2 \theta / (4\Omega)$. Using the flow equation (9) we find the spin-wave velocity contribution due to renormalization of the high frequency part of the spectrum

$$c(L) \sim \exp \left[-\frac{1+\theta}{2\theta} \left(\frac{L}{L_{\text{loc}}} \right)^{2(\theta+1)} \right], \quad (19)$$

which we have expressed in terms of the localization length (12) and used the relation $\theta_h = 1 + \theta$.

Order-disorder transition. – Above the lower critical dimension $d_{\text{lc}} = 4$ the quantum model (2) undergoes an order-disorder transition similar to that of the classical model [41]. For $\varepsilon < 0$ and $N > N_c$ the FRG equation (6) has a FP solution which is unstable in a single direction, and thus, describes the transition. For instance, the $O(3)$ and $O(4)$ RF models have the FPs with $\tilde{R}''(0) = -5.54|\varepsilon|$ and $\tilde{R}''(0) = -0.787|\varepsilon|$, respectively. For $N > 18$ the non analyticity of the RF FP becomes weaker than a linear cusp in $\tilde{R}''(\phi)$ and its value sticks to $\tilde{R}''(0) \approx -|\varepsilon|/(N-2)$. The large N behavior of the RA FP is given by $\tilde{R}''(0) \approx -|\varepsilon|(3N+40)/2(N-2)^2$.

The critical temperature $T_c(\Delta)$ is a function of the bare disorder strength (e.g. $\Delta = r^{(1)}$ for RF and $\Delta = r^{(2)}$ for RA) and vanishes at the quantum critical point $T_c(\Delta^*) = 0$. The only positive eigenvalue $\lambda = |\varepsilon|$ does not depend on N to one loop order and gives the critical exponent $\nu = 1/\lambda$, that describes the divergence of the correlation length in the classical regime $\xi \sim |T - T_c|^{-\nu}$ and at the quantum critical point $\xi \sim |\Delta - \Delta^*|^{-\nu}$. The hyperscaling relation between ν and the heat capacity exponent α is modified by the exponent θ as $\nu(d - \theta) = 2 - \alpha$. The averaged connected and disconnected correlation function exhibit a power-law behavior at the transition with the exponents η and $\bar{\eta}$ related by $\bar{\eta} = 2 + \eta - \theta$. The critical dynamics can be studied along the same lines

as for the dynamics in the QLRO phase. It turns out to be activated with the typical time $\tau \sim e^{C\xi^\theta/T}$ in the classical regime with $C = \Omega\rho_0\Lambda_0^{\theta-d+2}/K_d\theta$ and $\tau \sim e^{C'\xi^\Psi/\hbar^2}$ with $C' = 2C^2\theta/(1+\theta)$ and $\Psi = 2(\theta+1)$ at the quantum critical point. We expect that the latter relation holds also for the Ising case [20, 42].

Conclusion. – We have studied the dynamics of disordered interacting quantum rotors. The system is controlled by a quasi-classical zero temperature (*i.e.* infinite randomness) FP with an infinite dynamic critical exponent. Below the lower critical dimension $d_{lc} = 4$ the system has a quantum QLRO phase with a power-law decay of correlations. At zero temperature the spin-wave excitations are localized on the length scale L_{loc} . For $T > 0$ the spin-waves propagate via thermal activation over the energy barriers diverging in the thermodynamic limit. These results obtained for the 3D $O(2)$ RF and $O(3)$ RA models can be relevant for the quantum dynamics of the Bose glass and disordered quantum antiferromagnets. Above the lower critical dimension the system of quantum rotors undergoes an order-disorder phase transition with activated dynamics which is strongly suppressed in the vicinity of the quantum critical point.

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